Mechanism Design for Prediction Markets

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Mechanism **Design** for Prediction Markets

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Bet = Credible Opinion

• Q: Is Vinay Deolalikar’s proof of $P \neq NP$ correct?

“If Vinay Deolalikar is awarded the $1,000,000$ Clay Millennium Prize for his proof of $P \neq NP$, then I, Scott Aaronson, will personally supplement his prize by the amount of $200,000$.”

• Scott Aaronson: “I have a way of stating my prediction that no reasonable person could hold against me: I’ve literally bet my house on it.”
A Prediction Market

If I think that the probability of Obama being re-elected is $p$, I should

- **Buy** this security at any price less than $10p$
- **Sell** this security at any price greater than $10p$

Current price measures the population’s collective belief
Goal: Elicit and aggregate information about some event of interest.

The contract allows traders to express their information in a certain way.

Traders update their beliefs after observing the market price.
Evidence

• Racetrack odds beat track experts  [Figlewski 1979]
• I.E.M. beat political polls 451/596  [Forsythe 1992, 1999]
• HP market beat sales forecast 6/8  [Plott 2000]
• Sports betting markets provide accurate forecasts of game outcomes  [Gandar 1998][Thaler 1988]
  [Debnath EC’03][Schmidt 2002]
• Laboratory experiments confirm information aggregation  
  [Plott 1982;1988;1997][Forsythe 1990][Chen, EC’01]
Continuous Double Auction (CDA)

Barack Obama to be re-elected President in 2012
Last prediction was: $6.05 / share
Today's Change: $0.00 (+0%)
Contract Type: 0-100

Event: 2012 Presidential Election Winner (Individual)

Best (highest) price members are buying at

<table>
<thead>
<tr>
<th>Price</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6.02 / Share</td>
<td>18 shares</td>
</tr>
<tr>
<td>$6.01 / Share</td>
<td>198 shares</td>
</tr>
<tr>
<td>$6.00 / Share</td>
<td>74 shares</td>
</tr>
<tr>
<td>$5.99 / Share</td>
<td>113 shares</td>
</tr>
<tr>
<td>$5.98 / Share</td>
<td>13 shares</td>
</tr>
</tbody>
</table>

Best (lowest) price members are selling at

<table>
<thead>
<tr>
<th>Price</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6.04 / Share</td>
<td>5 shares</td>
</tr>
<tr>
<td>$6.05 / Share</td>
<td>36 shares</td>
</tr>
<tr>
<td>$6.06 / Share</td>
<td>113 shares</td>
</tr>
<tr>
<td>$6.07 / Share</td>
<td>311 shares</td>
</tr>
<tr>
<td>$6.08 / Share</td>
<td>201 shares</td>
</tr>
</tbody>
</table>
Design Objectives

• Liquidity
  – People can find counterparties to trade whenever they want.

• Bounded budget (loss)
  – Total loss of the market institution is bounded.

CDA can be illiquid but always satisfies bounded budget.
Hanson’s Automated Market Makers

Will the Chicago Bears beat the Green Bay Packers to advance to the Super Bowl?

Buy 25 shares

Change chance to: 53.40%
Change price to: $53.40
Available now: $4,061.19
Spend: $1,303.71

You will be buying 25 shares in addition to the 9 shares you already own.
Hanson’s Automated Market Makers
[Hanson 03, 07; Chen and Pennock 07; Chen and Vaughan 10]

- One contract for each outcome
  - Payments are determined by a cost potential function $C(q)$
    - $q_i$ is the current number of shares of the contract for outcome $i$ that have been purchased
    - Current cost of purchasing a bundle $r$ of shares is
      $$C(q + r) - C(q)$$
    - Instantaneous prices
      $$p_i(q) = \frac{\partial C(q)}{\partial q_i}$$
      Increasing in $q_i$, Sum to 1
Design Objectives

• Liquidity
  – People can find counterparties to trade whenever they want.

• Bounded budget (loss)
  – Total loss of the market institution is bounded.

Hanson’s market makers satisfy both liquidity and bounded budget.
Large Outcome Spaces

\[ n! \quad 2^n \quad \text{Infinite} \]

- Can not naively run Hanson’s market maker
- Calculating prices becomes intractable
  [Chen et al. 08]
- Humans are notoriously bad at assessing very small probabilities
Design Objectives

• Liquidity
  – People can find counterparties to trade whenever they want.

• Bounded budget (loss)
  – Total loss of the market institution is bounded.

• Computational tractability
  – The process of operating a market should be computationally manageable.

• Expressiveness
  – There are as few constraints as possible on the form of bets that people can use to express their opinions.
Expressiveness in Getting Information

• Things people can express today
  – A Democrat wins the election (with probability 0.65)
  – No bird flu outbreak in US before 2014
  – Microsoft’s stock price is greater than $30 by the end of 2012
  – Horse A will win the race

• Things people can not express very well today
  – A Democrat wins the election if he wins both Florida and Ohio
  – Oil price increases & US Recession in 2012
  – Microsoft’s stock price is between $30 and $35 by the end of 2012
  – Horse A beats Horse B
Expressiveness in Processing Information

• Today’s Independent Markets
  – Options at different strike prices
  – Horse race win, place, and show betting pools
  – Almost every market: Wall Street, Las Vegas, Intrade, ...

• Events are logically related, but independent markets let traders to make the inference

• What may be better
  – Traders focus on expressing their opinions in the way they want
  – The mechanism takes care of propagating information across logically related events
• **Given a set of contracts over a large (or infinite) outcome space, can we design a consistent market maker that has bounded budget and can be operated efficiently?**

<table>
<thead>
<tr>
<th>States</th>
<th>Contracts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A&lt;B</td>
</tr>
<tr>
<td>ABC</td>
<td>$1</td>
</tr>
<tr>
<td>ACB</td>
<td>$1</td>
</tr>
<tr>
<td>BAC</td>
<td>$0</td>
</tr>
<tr>
<td>BCA</td>
<td>$0</td>
</tr>
<tr>
<td>CAB</td>
<td>$1</td>
</tr>
<tr>
<td>CBA</td>
<td>$0</td>
</tr>
</tbody>
</table>
Inherent Challenge

$1 \iff A < B$

$1 \iff B < A$

$1 \iff A < C$

$1 \iff C < A$

$1 \iff B < C$

$1 \iff C < B$

\[ P_{A<B} + P_{A>B} = 1 \]

\[ P_{A<B} + P_{B<C} + P_{C<A} \leq 2 \]

???
Duality-Based Market Maker

[Abernethy, Chen, Vaughan 11, 12]

\[ C(\vec{q}) = \max_{\vec{x} \in \text{Hull}(\rho)} \vec{x} \cdot \vec{q} - R(\vec{x}) \]

- Liquidity
- Bounded budget
- Expressiveness (for the set of contracts)
- Can be tractable
Conclusion

• We consider the design of market mechanisms for information aggregation that satisfy
  – Liquidity
  – Bounded budget
  – Computational tractability
  – Expressiveness

• Many other important design objectives
  – “Truthfulness”
  – Liquidity sensitivity [Othman et. al, 10; Othman and Sandholm 12]
Thanks!

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